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THE MASS FUNCTION OF VOID GROUPS AS A PROBE OF THE PRIMORDIAL NON-GAUSSIANITY

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ABSTRACT

The primordial non-Gaussianity signal, if measured accurately, will allow us to distinguish between different candidate models for the cosmic inflation. Since the galaxy groups located in void regions are rare events, their abundance may be a sensitive probe of the primordial non-Gaussianity. We construct an analytic model for the mass function of void groups in the framework of the extended Press-Schechter theory with non-Gaussian initial conditions and investigate how it depends on the primordial non-Gaussianity parameter. A feasibility study is conducted by fitting the analytic mass function of void groups to the observational results from the galaxy group catalog of the Sloan Digital Sky Survey Data Release 4 with adjusting the primordial non-Gaussianity parameter.

Subject headings: cosmology:theory — large-scale structure of universe

1. INTRODUCTION

In precision cosmology, a fundamental task is to distinguish between various candidate models for the cosmic inflation. One way to perform this task is to measure the degree of non-Gaussianity in the primordial density field. Although the primordial density field is regarded as nearly Gaussian in all inflationary scenarios (Guth & Pi 1982), the degree of its non-Gaussianity differs between the models. For instance, in the single-field slow-roll inflationary model, the deviation from Gaussianity is small enough to be unobservable (Verde et al. 2001). Meanwhile in some multi-field inflationary models the non-Gaussianity can be generated to a detectable level (Battefeld & Easther 2007, and references therein). For a thorough review on the predictions of various inflationary models for the primordial non-Gaussianity, see Bartolo et al. (2004)

The cosmic microwave background radiation (CMB) is a useful probe of the primordial non-Gaussianity as it reflects the linear density perturbations. Yet, if the primordial non-Gaussianity is scale-dependent, the constraint from the CMB analysis is rather restricted to the very large-scale (Verde et al. 2001; Komatsu & Spergel 2001). The large-scale structure is a powerful alternative probe of the primordial non-Gaussianity on the sub-CMB scale. It has been well known that the abundance of high-z clusters are rare enough to constrain the primordial non-Gaussianity (Lucchin & Matarrese 1988; Chiu et al. 2998; Matarrese et al. 2000; Weinberg & Kamionkowski 2003; Carbone et al. 2008; LoVerde et al. 2008; Grossi et al. 2009). This probe, however, is likely to suffer from large systematics involved in the inaccurate measurement of the masses of high-z clusters. The abundance of cosmic voids is another probe of the primordial non-Gaussianity based on the large-scale structure (Kamionkowski et al. 2009). One difficulty in using this probe lies in the fact that there is no unique way to define voids (Carbone et al. 2008). Recently, it has been claimed that using the clustering properties of highly biased large-scale structures the primordial non-Gaussianity parameter can be measured with accuracy as high as the one obtained from the CMB analysis (Carbone et al. 2008; Slosar et al. 2008; Jeong & Komatsu 2009).

Here, we propose the mass function of present galaxy groups embedded in void regions as a new probe of the primordial non-Gaussianity. It is not only the high-z clusters but also the low-z void groups that are so rare that their abundance may depend sensitively on the initial conditions. Furthermore, the mass estimation of low-z galaxy groups should be much more reliable than that of high-z clusters (Yang et al. 2007). Throughout this Letter, we assume a WMAP 5 cosmology (Dunkley et al. 2009).

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2. AN ANALYTIC MODEL

The Press-Schechter theory (Press & Schechter 1974, PS hereafter) provides an analytic framework within which the number density of bound objects as a function of mass, $dN_{\rm PS}/dM$, can be obtained:

$$\frac{dN}{dM} = A\frac{\bar{\rho}}{M}\frac{d}{dM}\left|\int_{\delta_c(z)}^{\infty} p(\delta_M)d\delta\right|, \tag{1}$$

where $\bar{\rho}$ is the mean background density, $\delta_c(z)$ is the critical density contrast for gravitational collapse at redshift z, $p(\delta_M)$ is the probability density distribution of the density field δ_M smoothed on the mass scale of M, and A is the normalization constant. Basically, equation (1) states that the number density of bound objects can be inferred from the differential volume fraction occupied by those regions in the linear density field whose average density contrast δ_M reaches a certain threshold, δ_c. In the original PS theory, the initial density field is assumed to be Gaussian as $p(\delta_M) = \exp\left[-\delta_M^2/(2\sigma_M^2)\right]/(\sqrt{2\pi}\sigma_M)$.

Here σ_M is the rms density fluctuation smoothed on the mass scale M and $\delta_c \equiv \delta_{c0}/D_+(z)$ where δ_{c0} is the critical density contrast at $z=0$ and $D_+(z)$ is the linear growth factor. For a WMAP 5 cosmology, we find $\delta_{c0} \approx 1.62$. Note that the mass function of bound objects depends on the initial conditions through its dependence

on σ_M which is a function of the density parameter, Ω_m and the amplitude of the linear power spectrum, σ_8.

Now, let us consider the case of non-Gaussian initial conditions that is often characterized by the primordial non-Gaussianity parameter $f_{\rm NL}$ as $\psi(\mathbf{x}) = \phi(\mathbf{x}) + f_{NL}\left(\phi^2(\mathbf{x}) - \langle\phi^2(\mathbf{x})\rangle\right)$ where ϕ is a Gaussian random field and $\psi(\mathbf{x})$ is the linearly extrapolated gravitational potential at $z = 0$ (e.g., LoVerde et al. 2008; Grossi et al. 2009). The functional form of $p(\delta_M)$ for the non-Gaussian case will be in general different from the Gaussian case. However, provided that the degree of the non-Gaussianity is very small and scale independent, $p(\delta_M)$ has the same functional form as the Gaussian case at first order (Lucchin & Matarrese 1988; Matarrese et al. 2000; Verde et al. 2001). The only difference is the value of the critical density contrast, δ_{c*}, which is related to that of the Gaussian case, δ_c as

$$\delta_{c*}(z) = \delta_c(z)\left[1 - \frac{S_3}{3}\delta_c(z)\right]^{1/2}. \tag{2}$$

Here S_3 is a skewness parameter, related to the primordial non-Gaussianity parameter $f_{\rm NL}$ as $S_3 = \frac{3}{5}f_{\rm NL}\mu_3^{(1)}/(\mu_2^{(1)})^2$ where $\mu_2^{(1)}$ and $\mu_3^{(1)}$ denote the variance and skewness of the smoothed non-Gaussian density field at first order, respectively (see eqs.43-45 in Matarrese et al. 2000). Therefore, the PS mass function with non-Gaussian initial conditions is a function of M and $f_{\rm NL}$: $dN_{\rm PS}(f_{\rm NL}, M)/dM$. The case of $f_{\rm NL} = 0$ corresponds to the original one with Gaussian initial conditions.

On the group scale, $dN_{\rm PS}(f_{\rm NL}, M)/dM$ does not change sensitively with the initial conditions since the galaxy groups are not rare events. However, those groups located in voids should be so rare that their abundance may depend sensitively on the initial conditions. Before deriving the abundance of void groups, we clarify the meaning of a *void region*. Following Hahn et al. (2007), we define a void region on mass scale M as a region where the three eigenvalues of the tidal tensor at a given region on mass scale M are less than zero. Then, we replace $p(\delta_M)$ in eq. (1) by $p(\delta_M|\lambda'_{1M'} < 0)$ which represents the conditional probability density distribution that the density contrast has a certain value on the mass scale M provided that the largest eigenvalue of the tidal tensor is negative on some larger mass scale $M' > M$. From here on, δ denotes the density contrast on the mass scale M, while λ'_1, λ'_2, λ'_3 are the three eigenvalues of the tidal field on some larger mass scale M'.

The joint probability distribution $p(\delta, \lambda'_1, \lambda'_2, \lambda'_3)$ for the case of Gaussian initial conditions has been already derived by Lee (2006) as

$$p(\delta, \lambda'_1, \lambda'_2, \lambda'_3) = \frac{1}{\sqrt{2\pi}\sigma_\Delta}\frac{3375}{8\sqrt{5}\pi\sigma'^6}\exp\left[-\frac{(\delta - I'_1)^2}{2\sigma_\Delta^2}\right]\times \exp\left(-\frac{3I_1'^2}{\sigma'^2} + \frac{15I'_2}{2\sigma'^2}\right)(\lambda'_1 - \lambda'_2)(\lambda'_2 - \lambda'_3)(\lambda'_1 - \lambda'_3), \tag{3}$$

with $\sigma_\Delta^2 \equiv \sigma^2 - \sigma'^2$, $I'_1 = \lambda'_1 + \lambda'_2 + \lambda'_3$, and $I'_1 = \lambda'_1\lambda'_2 + \lambda'_2\lambda'_3 + \lambda'_1\lambda'_3$. Here σ and σ' represents the rms density fluctuations on the mass scale M and M', respectively. The conditional probability density, $p(\delta|\lambda'_1 < 0)$, is now written as

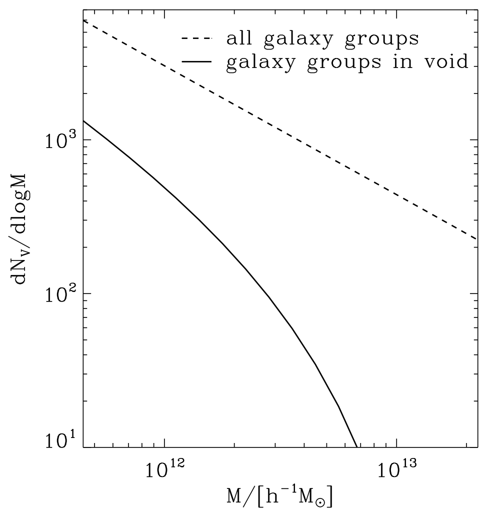

FIG. 1.— Mass function of void groups (solid line) and of all groups (dashed line) in the extended Press-Schechter formalism.

$$p(\delta|\lambda'_1 < 0) = \frac{\int_{-\infty}^{0} d\lambda_1 \int_{-\infty}^{\lambda'_1} d\lambda_2 \int_{-\infty}^{\lambda'_2} d\lambda_3 p(\delta, \lambda'_1, \lambda'_2, \lambda'_3)}{\int_{-\infty}^{0} d\lambda_1 \int_{-\infty}^{\lambda'_1} d\lambda_2 \int_{-\infty}^{\lambda'_2} d\lambda_3 p(\lambda'_1, \lambda'_2, \lambda'_3)}. \tag{4}$$

Putting $p(\delta|\lambda'_1 < 0)$ in eq. (1), we evaluate the mass function of void groups with Gaussian initial conditions, $dN_{\rm V}^{\rm G}/dM$.

Using the methodology suggested by LoVerde et al. (2008), we model the mass function of void groups with non-Gaussian initial conditions, $dN_{\rm V}^{\rm NG}/dM$, as

$$\frac{dN_{\rm V}^{\rm NG}}{dM} = \frac{dN_{\rm V}^{\rm G}}{dM}\frac{dN_{\rm PS}(f_{\rm NL}, M)/dM}{dN_{\rm PS}(f_{\rm NL} = 0, M)/dM}, \tag{5}$$

where $dN_{\rm PS}(f_{\rm NL} = 0, M)/dM$ and $dN_{\rm PS}(f_{\rm NL}, M)/dM$ represents the PS mass function with Gaussian and non-Gaussian initial conditions, respectively (see eq.[4.20] in Loverde et al. 2009). Figure 1 plots the mass function of void groups (solid line), which normalized as $\int dN_V/d\log M_i = N_{vg}$ where N_{vg} is the total number of void groups found in the observational data (see §3.) The mass function of all groups (dashed line) are also plotted for comparison. As can be seen, the mass function of void groups decreases very rapidly with mass, which indicates that it must depend sensitively on the initial conditions.

It has to be mentioned here that equation (5) has yet to be validated against numerical results. The methodology of LoVerde et al. (2008) that equation (5) is based on has recently been tested against N-body simulations and found to be valid in the high-mass sections (Grossi et al. 2009). Yet, to fully justify the use of equation (5) for the evaluation of the abundance of void groups with non-Gaussian initial conditions, it will be required to test equation (5) numerically on the group-mass scale. Furthermore, for the case of scale-dependent non-Gaussianity the mass function of void groups would have much more complicated even at first order (LoVerde et al. 2008). The focus of this work, however, is on the proof of a concept that the abundance of present galaxy groups in voids can be used as a probe of

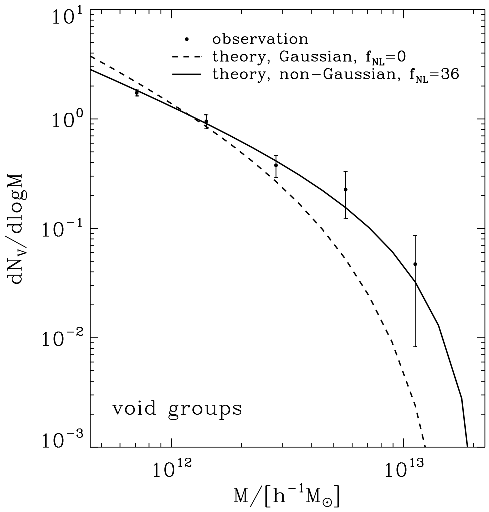

FIG. 2.— Mass function of void groups with the best-fit value of $f_{\rm NL}$ (solid line) determined by fitting to the observational data from the SDSS DR4 (dots). The errors include both the Poisson noise and the cosmic variance. The mass function of void groups for the Gaussian case ($f_{\rm NL} = 0$) is also plotted for comparison (dashed).

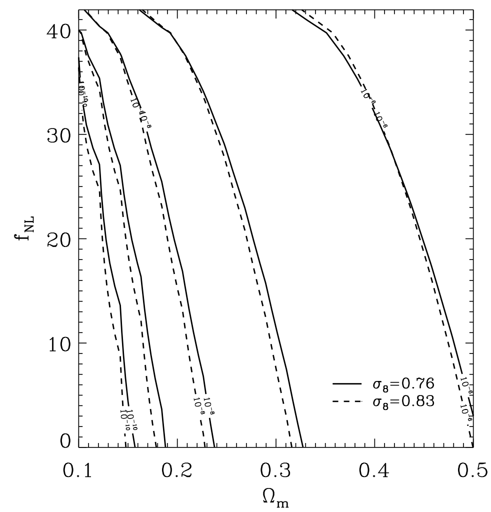

FIG. 3.— Degeneracy curves in the $f_{\rm NL}$-$\Omega_{\rm m}$ plane with the value of σ_8 set at 0.76 (solid)and at 0.83 (dashed).

the primordial non-Gaussianity. Henceforth, here we use equation (5) as an ansatz and consider only the scale-independent case for simplicity.

3. A FEASIBILITY STUDY

We conduct a feasibility study by comparing the analytic mass function of void groups obtained in §2 with the observational result from the galaxy group catalog of the Sloan Digital Sky Survey Data Release 4 provided by (Yang et al. 2007) who measured the masses of the SDSS galaxy groups were estimated from the characteristic luminosity (or stellar masses) using the WMAP 3 cosmology (Spergel et al. 2007). To identify void groups from the SDSS group catalog, we use the real-space tidal field reconstructed by Lee & Erdogdu (2007) on 64^3 pixels in a box of linear length $400h^{-1}$Mpc from the Two Mass Redshift survey (2MRS) (Huchra et al. 2005). We smooth the 2MRS tidal field with a Gaussian filter of scale radius $8h^{-1}$Mpc on which scale the density field is still in the quasi-linear regime, calculate the three eigenvalues of the smoothed tidal field at each pixel and mark as voids those pixels in which all three eigenvalues are less than zero. A total of 550 galaxy groups at redshifts $0.01 \le z < 0.04$ in mass range of $11.7 \le \log M/(h^{-1}M_\odot) \le 13.4$ are found in the void pixels. Binning their mass range in the logarithmic scale, we measure their abundance, $dN_V/d\log M$, which is renormalized to be $\int dN_V = 1$. To account for the cosmic variance as well as the Poisson noise in the measurement of $dN_V/d\log M$, we divide the selected void groups into 6 subsamples and measure $dN_V/d\log M$ for each subsample separately. We calculate the jackknife errors as the standard deviation in the measurement of the mean $dN_V/d\log M$ averaged over the 6 subsamples at each mass bin.

We fit the observational result to the analytic model by adjusting the value of $f_{\rm NL}$. To account for the correlations between the mass bins, we employ the *generalized* χ^2-statistics to determine the best-fit value of $f_{\rm NL}$: $\chi^2 = [n_i - n(\log M_i; f_{\rm NL})]C_{ij}^{-1}[n_i - n(\log M_i; f_{\rm NL})]$ where $n_i \equiv dN/d\log M_i$ and $n(\log M_i; f_{\rm NL})$ represents the observational and analytical results evaluated at the i-th logarithmic mass bin, $\log M_i$, respectively. And (C_{ij}) is the covariance matrix defined as $C_{ij} = \langle (n_i - n_{0i})(n_j - n_{0j})\rangle$ where n_{0i} represents the mean of n_i averaged over all samples. Finally, the uncertainty in the measurement of $f_{\rm NL}$ is calculated as the curvature of the χ^2 function at the minimum. Through this fitting procedure, we find $f_{\rm NL} = 36 \pm 1$. Figure 2 plots the observational results (dots) and the analytic model with the best-fit value of $f_{\rm NL} = 36$ (solid line). The median redshift of the SDSS void groups, $z = 0.03$, is used for the value of z in the analytic model. For comparison, the analytic model with $f_{\rm NL} = 0$ is also plotted (dotted line). As can be seen, the observational results agree better with the analytic model with non-Gaussian initial conditions.

It is, however, expected that there is a degeneracy between $f_{\rm NL}$ and the other key cosmological parameters on which the mass function of void groups depend. Here, we investigate the degeneracy between Ω_m and $f_{\rm NL}$, setting σ_8 at the value determined by WMAP 5 cosmology. Varying the values of Ω_m and $f_{\rm NL}$, we recalculate $dN_V/d\log M$ at a typical group mass scale of $M = 10^{13}h^{-1}M_\odot$. Figure 3 plots a family of the degeneracy curves in the Ω_m-$f_{\rm NL}$ plane with the value of σ_8 set at 0.76 (solid) and 0.83 (dashed). As can be seen, a strong degeneracy exists between the two parameters. For a fixed value of $dN_V/d\log M$, the value of $f_{\rm NL}$ increases as the value of Ω_m decreases. The comparison between the solid and the dashed lines indicate that the degree of the degeneracy between Ω_m and $f_{\rm NL}$ increases as the value of σ_8 increases. It also indicates that if Ω_m is fixed, the value of $f_{\rm NL}$ increases as the value of σ_8 decreases. To break this parameter degeneracy, it will be necessary to combine our analysis with other analyses.

The preliminary results of this feasibility study, however, are subject to several caveats. First, the analytic model assumes the scale-independent Gaussianity. To be more realistic, it is necessary to account for the scale-dependent non-Gaussianity. The second caveat lies in the limitation of the Press-Schechter formalism. As shown by several authors (Lee & Shandarin 1998; Sheth & Tormen

1999; Jenkins et al. 2001), the real gravitational process deviates from the spherical dynamics on which the PS mass function is based. It has to be tested how significantly the deviation of gravitational collapse process from the spherical dynamics affects the abundance of void groups. The third caveat comes from the fact that the validity of equation (5) has yet to be confirmed. Although Grossi et al. (2009) have shown that this methodology suggested by LoVerde et al. (2008) to model departures from non-Gaussianity leads to an excellent approximation on the cluster scale, it has to be confirmed by N-body simulations whether the same methodology can be used to count the number of void groups with non-Gaussian initial conditions. Fourth, the different mass-to-light ratios of the void galaxies from that of the wall galaxies has to be taken into account. According to Rojas et al. (2005), the specific star formation rate in void galaxies is higher than that in wall galaxies, suggesting that the mass of the void groups in the SDSS Galaxy group catalog are likely to be overestimated. Fifth, Yang et al. (2007) measured the masses of SDSS galaxy groups assuming the old WMAP 3 cosmology (Spergel et al. 2007). It will be necessary to use the values of the most updated WMAP 5 parameters for the more accurate calculation of the masses of void groups.

As a final conclusion, we have proved that the abundance of void groups can in principle be a useful probe of the primordial non-Gaussianity parameter. For a robust probe, however, it will be required to refine further the analytic model of the abundance of void groups and to improve the mass estimation of galaxy groups in void regions, which is the direction of our future work.

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